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10EC046

M.Tech. Degree Examination, December 2011
Linear Algebra

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. If A is an $m \times n$ matrix and $m < n$, then prove that the homogeneous system of linear equations $AX = 0$ has a non – trivial solution. **(04 Marks)**

b. Find for all values of 'k' the system of equations
$x + y + z = 1$
$x + 2y + 4z = k$
$x + 4y + 10z = k^2$
posseses a solution. Solve completely in each case. **(08 Marks)**

c. Solve the system of equations
$x_1 + 2x_2 + x_3 = 0$
$2x_1 + 2x_2 + 3x_3 = 3$
$-x_1 - 3x_2 = 2$
by LU – factorization with $u_{ii} = 1$. **(08 Marks)**

2 a. Prove that a non – empty subset W is a subspace of a vector space V over F, if and only if $a\alpha + b\beta \in W\ \forall \alpha, \beta \in W$ and $a, b \in F$. **(07 Marks)**

b. If W_1 and W_2 are subspaces of the vector space V(F) then show that show that $W_1 + W_2$ is a subspace of V(F). **(07 Marks)**

c. Show that the set S = {(1, 0, 1), (1, 1, 0), (–1, 0, –1)} is linearly dependent in $V_3(R)$. **(06 Marks)**

3 a. Show that the set B = = {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is a basis of the vector space $V_3(R)$. **(04 Marks)**

b. Show that a mapping $T : U \to V$ from the vector space U(F) in V(F) is a linear transformation iff $T(c_1\alpha + c_2\beta) = c_1 T(\alpha) + c_2 T(\beta)\ \forall c_1, c_2 \in F$ and $\alpha, \beta \in U$. **(06 Marks)**

c. If $T : U \to V$ is linear transformation, then, show that
i) $T(0) = O'$ where O and O′ are zero vectors of U and V respectively
ii) $T(-\alpha) = -T(\alpha)\ \forall \alpha \in U$
iii) $T(c_1\alpha_1 + c_2\alpha_2 + \ldots + c_n\alpha_n) = c_1T(\alpha_1) + c_2T(\alpha_2) + \ldots + c_nT(\alpha_n)$
iv) $T(\alpha - \beta) = T(\alpha) - T(\beta)$. **(10 Marks)**

4 a. Give the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ determine the linear transformation $T : V_3(R) \to V_2(R)$ and relative to the bases B_1 and B_2 given by
i) B_1 and B_2 are the standered bases of $V_3(R)$ and $V_2(R)$ respectively
ii) $B_1 = \{(1, 1, 1), (1, 2, 3), (1, 0, 0)$
$B_2 = \{(1, 1), (1, -1)\}$. **(10 Marks)**

b. State and prove the Rank – nullity theorem. **(10 Marks)**

5 a. If $T : V \rightarrow W$ be a non – singular linear mapping, then prove that $T^{-1} : W \rightarrow V$ is linear and bijective. **(06 Marks)**

b. Prove that every vector space V over the real field R of dimension n is isomorphic $V_n(R)$. **(08 Marks)**

c. If T_1 and T_2 be linear operators on R^2 defined as follows $T_1(x_1, x_2) = (x_2, x_1)$, $T_2(x_1, x_2) = (x_1, 0)$ then show that. $T_1T_2 \neq T_2T_1$. **(06 Marks)**

6 a. If T be a linear operator on $V_3(R)$ defined by
$T(a, b, c) = (3a, a - b, 2a + b + c)\ \forall\ a, b, c, \in V_3(R)$ then prove that $(T^2 - I)(T - 3I) = 0$. **(10 Marks)**

b. Let T be a linear operator on a finite dimensional vector space V. If f is the characteristics polynomial for T, then f(T) = 0; in other words, the minimal polynomial divides the characteristic polynomial for T. **(10 Marks)**

7 a. Define inner product space. Then show that $u = (u_1, u_2)$, $v = (v_1, v_2)$ in R^2 defined by $\langle u, v\rangle = 4u_1v_1 + 5u_2v_2$ is inner product space. **(07 Marks)**

b. $S = \{u_1, u_2, \ldots, u_p\}$ is an orthogonal set of non zero vectors in R^n. Then show that S is linearly independent and hence is a basis for the subspace spanned by S. **(08 Marks)**

c. Find and QR – factorization of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(05 Marks)

8 a. Convert the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ in to a quadratic form with no – cross product. **(10 Marks)**

b. Find the maximum value of $Q(x) = 5x_1^2 + 6\ x_2^2 + 7\ x_3^2 + 4x_1x_2 - 4x_1x_3$ subjected to constrain $x^T x = 1$. **(10 Marks)**

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